University of California, Berkeley
Physics H7C Fall 2002 (*Strovink*)

PROBLEM SET 5

1.
Two identical horizontal very thin slits in a black plate in the plane $z = 0$ are centered at $y = \pm\frac{h}{2}$, where y is the vertical coordinate. A screen with vertical coordinate Y is located a distance D downstream. If an analyzer is present, it is located just upstream of the screen. Fraunhofer conditions apply, *i.e.* $kh^2 \ll D$, and small-angle approximations can be made, *i.e.* $|y| \ll D$, $|Y| \ll D$. Plane wave A is normally incident on the top slit and plane wave B is normally incident on the bottom slit, with

$$\vec{E}_A \propto \Re(\hat{x}\, e^{i(kz-\omega t)})$$
$$\vec{E}_B \propto \Re(\hat{y}\, e^{i(kz-\omega t)})\ ,$$

with $\Re$ denoting the real part. When either slit is blocked and no analyzer is in place, the irradiance $I(Y = 0) \equiv I_0$. When neither slit is blocked, find $I(Y)/I_0$, where I_0 is defined above, for the following cases:
(a.)
No analyzer is in place.
(b.)
The analyzer accepts only light polarized along $\sqrt{\frac{1}{2}}(\hat{x} + \hat{y})$.
(c.)
The analyzer accepts only light polarized along $\sqrt{\frac{1}{2}}(\hat{x} - \hat{y})$.

2.
A Fabry-Perot etalon (Fowles 4.1-4.3) is fashioned from a single slab of transparent material having a high refractive index ($n = 4.5$) and a thickness of 2 cm. The surfaces of the slab have a reflectance $|r|^2$ of 0.90. If the etalon is used in the vicinity of wavelength 546 nm (Hg green line), determine
(a.)
the order of interference;
(b.)
the ratio $T_{\max}/T_{\min}$, where $T = |t|^2$ is the transmittance of the etalon;
(c.)
the resolving power $\lambda/|\Delta\lambda|$ of the etalon.

3.
At normal incidence, show that depositing a $\frac{\lambda}{2}$ coating *of any index* on a glass plate will leave the plate's reflectance $|r|^2$ unchanged.

4.
Consider the following practical solution for coating a plate made of glass ($n = 1.52$) to suppress reflections at a particular wavelength λ. Light passes from vacuum at normal incidence through two layers of coating into the glass. The first layer is $\frac{\lambda}{4}$ of CeF_3 ($n = 1.65$); the second is $\frac{\lambda}{4}$ of ZrO_2 ($n = 2.1$). Show that the resulting reflectance $|r|^2$ is only of order 0.1%.

5.
Prove that

$$\sum_{n=1}^{N} \exp(i\phi_n) = \frac{\sin N\Delta\phi/2}{\sin\Delta\phi/2} \exp(i\bar{\phi})\ ,$$

where

$$\Delta\phi \equiv \phi_{n+1} - \phi_n\ ,$$

and $\bar{\phi}$ is the average of the ϕ_n.

6.
A collimated light beam is incident normally on three very narrow, identical, equally spaced slits. At the center of the pattern projected on a faraway screen, the irradiance is $I_{\max}$.
(a.)
If the irradiance I_P at some point P on the screen is zero, what is the phase difference between light arriving at P from neighboring slits?
(b.)
If the phase difference between light waves arriving at P from neighboring slits is π, determine the ratio $I_P/I_{\max}$.
(c.)

If the average irradiance on the entire screen is I_{av}, what is the ratio I_P/I_{av} at the central maximum?

7.

A double-slit interference pattern is formed using Hg green light at 546 nm. Each slit has a width of 0.1 mm. The pattern reveals that the fourth-order interference maxima are missing from the pattern.

(a.)

What is the slit separation?

(b.)

What are the irradiances of the first three orders of interference fringes, relative to the zeroth-order maximum?

8.

The century-old telescope at Lick Observatory atop Mt. Hamilton near San Jose (readily accessible by bicycle starting from Fremont BART) is still among the largest refracting telescopes in the world. (For a fixed focal length, the largest lens diameter produces the smallest-diameter Airy disk and Airy rings, as in Fowles' Fig. 5.13.) The Lick refractor has a diameter of 3 ft and a focal length of 56 ft. Determine the radii of the first and second bright rings surrounding the Airy disk in the diffraction pattern formed by a star on the focal plane. (The first two secondary maxima of the function $J_1(\rho)/\rho$, where J_1 is the first-order Bessel function of the first kind, occur at $\rho = 5.14$ and $\rho = 8.42$.)